

VOLUME

A handout from The Learning Center at Trident Technical College

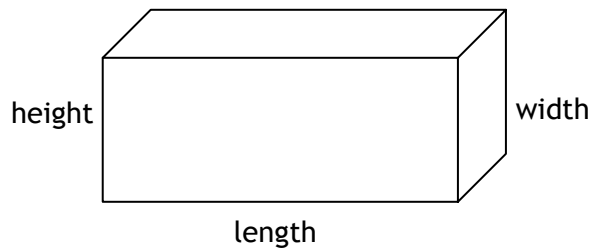
Volume of a solid geometric figure is the amount of the space it occupies, measured in terms of three dimensions (length, width, and height).

Volume units are cubic inches (in^3 or cu in), cubic feet (ft^3 or cu ft), cubic yards (yd^3 or cu yd), and cubic meters (m^3).

Below are some formulas related to single geometric figures such as rectangular solids, cylinders, cubes, and spheres.

Rectangular Solid

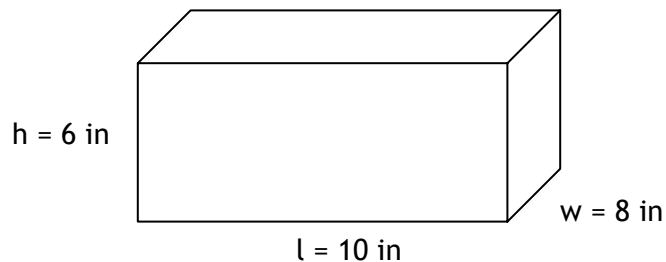
$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$



Volume of a rectangular solid (box) is shown by multiplying the length times width times height. Volume is expressed in cubic units.

Example 1: Find the volume of a box that is 10 inches long, 8 inches wide, and 6 inches deep.

Solution:



$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

$$V = (10 \text{ in})(8 \text{ in})(6 \text{ in})$$

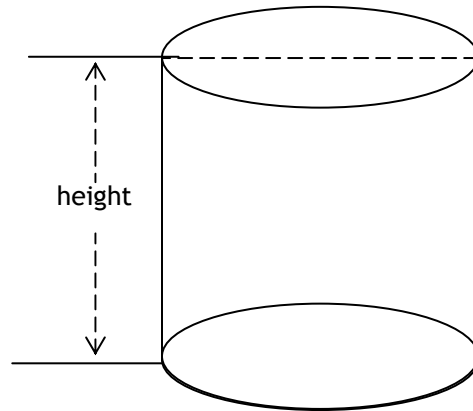
$$V = 480 \text{ in}^3$$

Therefore, the volume is 480 in^3 . The answer may also be written as 480 cubic inches.

Cylinders

Volume = $\pi \times \text{radius squared} \times \text{height}$

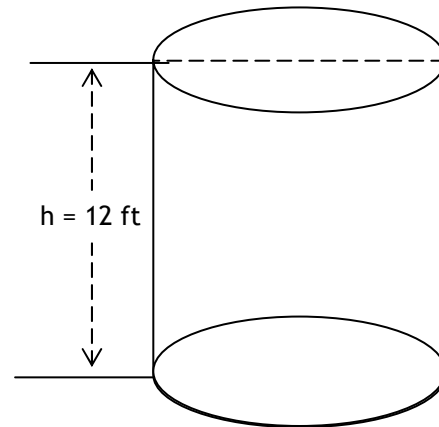
$$V = \pi r^2 h$$



Cylinders have two circular bases that are the same size and parallel. The height of a cylinder is the distance between the bases. The volume is equal to π times the radius squared times the height. The volume is expressed in cubic units.

Example 2: How many gallons will a cylinder hold if it has a radius of 3.5 feet and a height of 12 feet? (1 cu ft = 7.5 gal)

(Use $\frac{22}{7}$ for π)



Strategy: To find the number of gallons a cylinder will hold:

- Use the formula for the volume of a cylinder.
- Convert the volume to gallons

Solution:

Volume = $\pi \times \text{radius}^2 \times \text{height}$

$$V = \pi r^2 h$$

$$V = \frac{22}{7} \times \frac{(3.5 \text{ ft})^2}{1} \times \frac{12 \text{ ft}}{1}$$

$$V = \frac{22}{7} \times \frac{12.25 \text{ ft}}{1} \times \frac{12 \text{ ft}}{1}$$

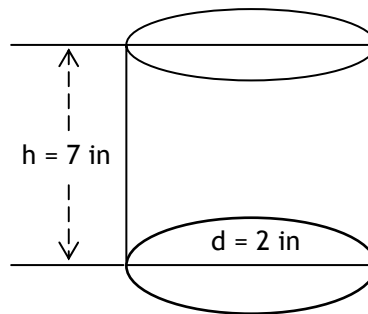
$$V = \frac{3234 \text{ ft}}{7}$$

$$V = 462 \text{ ft}^3$$

$$462 \text{ cu ft} \times \frac{7.5 \text{ gal}}{1 \text{ cu ft}} = 3465 \text{ gal}$$

Therefore, the cylinder will hold 3465 gallons.

Example 3: Find the capacity of a can of tomato juice which has a diameter of 2 inches and a height of 7 inches. Use $\frac{22}{7}$ for π .



Strategy:

- To find the volume of a cylinder whose diameter is given, use the formula (radius = $\frac{1}{2}$ diameter) before finding the volume.

Solution:

$$\text{radius} = \frac{1}{2} \text{ diameter}$$

$$r = \frac{1}{2} \times \frac{2 \text{ in}}{1}$$

$$r = \frac{2 \text{ in}}{2}$$

$$r = 1 \text{ in}$$

$$\text{Volume} = \pi r^2 h$$

$$V = \frac{22}{7} \times \frac{(1)^2}{1} \times \frac{7 \text{ in}}{1}$$

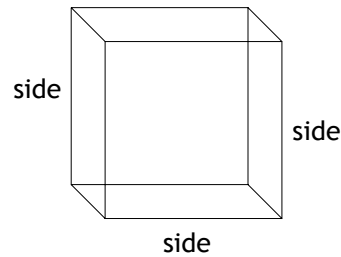
$$V = \frac{154}{7}$$

$$V = 22 \text{ in}^3$$

Therefore, the volume is 22 in^3 . The answer may also be written as 22 cubic inches.

Cube

$$\text{Volume} = S \times S \times S = S^3$$



The volume of a cube is the space enclosed within a three-dimensional figure. Volume is measured in cubic units. It is read "volume equals sides cubed." To cube a number multiply it times itself three times.

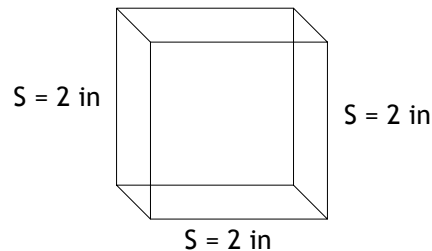
Example 4: Find the volume of a cube that has 2-inch sides.

Solution:

$$\text{Volume} = S^3$$

$$V = (2 \text{ in})^3$$

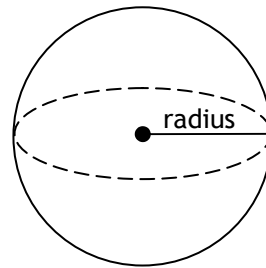
$$V = 8 \text{ in}^3$$



Therefore, the volume is 8 in^3 . The answer may also be written as 8 cubic inches.

Sphere

$$\text{Volume} = \frac{4}{3} \pi r^3$$



The volume of a sphere is found by multiplying four-thirds times pi (π) times the radius cubed.

Example 5: Find the volume of a sphere with a 11-mm radius.
(Use 3.14 for π .) Round to nearest hundredth.

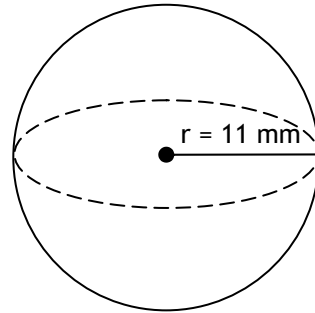
$$\text{Volume} = \frac{4}{3} \pi r^3$$

$$V = \frac{4}{3} \times \frac{3.14}{1} \times \frac{(11\text{mm})^3}{1}$$

$$V = \frac{4}{3} \times \frac{3.14}{1} \times \frac{1331\text{mm}}{1}$$

$$V = \frac{16717.36\text{mm}}{3}$$

$$V = 5572.45\text{mm}^3$$



Therefore, the volume is 5572.45mm^3 . The answer may also be written as 5572.45 cubic millimeters.

Example 6: Find the volume of a sphere with a 6 ft diameter. Use 3.14 for π .

Strategy:

To find the volume of a sphere when the diameter is given, use the formula (radius = $\frac{1}{2}$ diameter) before finding the volume.

Solution:

$$\text{radius} = \frac{1}{2} \text{ diameter}$$

$$r = \frac{1}{2} \times \frac{6\text{ft}}{2}$$

$$r = \frac{6\text{ft}}{2}$$

$$r = 3\text{ft}$$

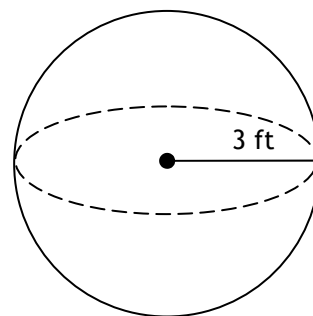
$$\text{Volume} = \frac{4}{3} \pi r^3$$

$$V = \frac{4}{3} \times \frac{3.14}{1} \times \frac{(3\text{ft})^3}{1}$$

$$V = \frac{4}{3} \times \frac{3.14}{1} \times \frac{27\text{ft}}{1}$$

$$V = \frac{339.12\text{ft}}{3}$$

$$V = 113.04\text{ft}^3$$

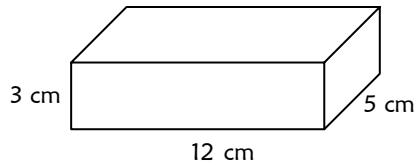


Therefore, the volume is 113.04ft^3 . The answer may also be written as 113.04 cubic feet.

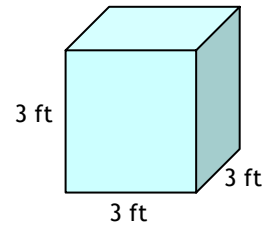
PRACTICE:

Find the volume of the given figures. Use 3.14 for π .

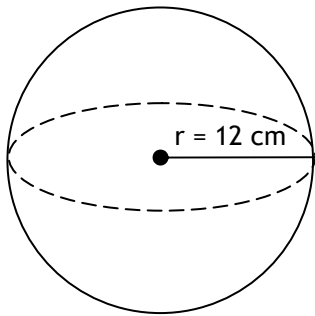
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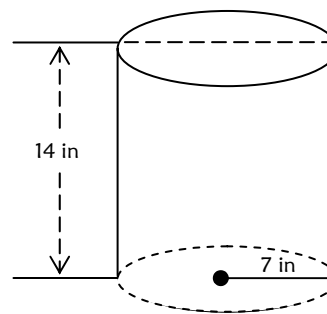
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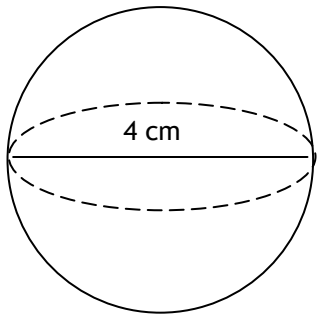
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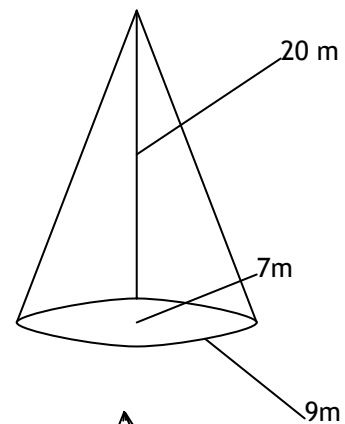
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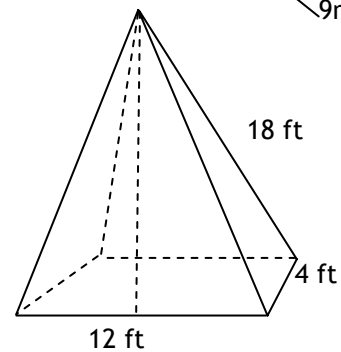
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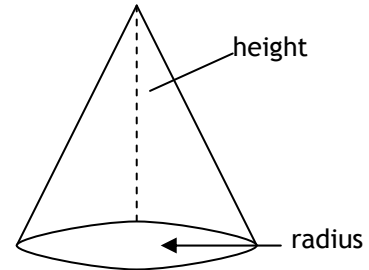
Answers:

1. 180 cm^3
2. 27 ft^3
3. 7234.56 cm^3
4. 2154.04 in^3
5. 33.49 cm^3
6. 1695.6 m^3
7. 288.000 ft^3

7.



Right Circular Cone



$$\text{Volume} = \frac{1}{3} \pi \times \text{radius}^2 \times \text{height}$$

Volume of a right circular cone is a cone whose base is a circle and whose apex is on a line perpendicular to the plane containing the base. The apex is equidistant from all points on the circle.

Example 4: Find the volume of a right circular cone if its radius is 10 centimeters and its height is 20 centimeters. (Use 3.14 for π). Round to the nearest tenth.

Solution:

$$\text{Volume} = \frac{1}{3} \pi \times \text{radius}^2 \times \text{height}$$

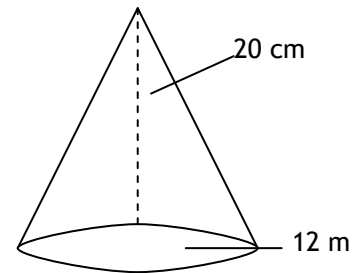
$$V = \frac{1}{3} \pi r^2 h$$

$$V = 1 \times 3.14 \times (10\text{cm})^2 \times 20\text{cm}$$

$$V = 1 \times 3.14 \times \frac{100\text{cm}^2 \times 20\text{cm}}{3}$$

$$V = \frac{6280\text{cm}^3}{3}$$

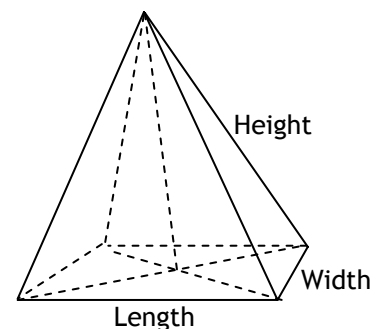
$$V = 2093.3 \text{ cm}^3$$



Therefore, the right circular cone is 2093.3 cm³. The answer may also be written as 2093.3 cubic centimeters.

Rectangular Pyramid

$$\text{Volume} = \frac{1}{3} \times \text{length} \times \text{width} \times \text{height}$$



A pyramid is a space figure with a square base and 4 triangle- shaped sided.

Volume of a rectangular pyramid is shown by multiplying one-third ($1/3$) times the length times width time height. Volume is a expressed in cubic units.

Example 5: Find the volume of a rectangular pyramid with the length 18 cm, width 10 cm, and height 3cm.

Solution:

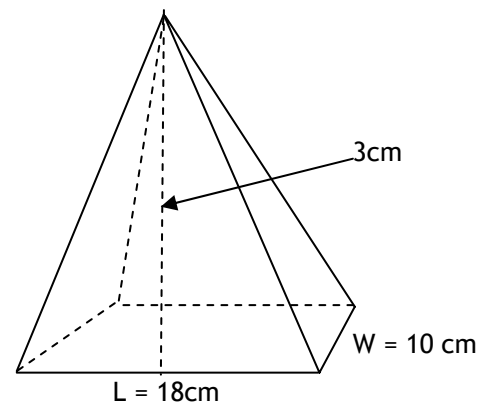
Volume = $1/3 \times \text{length} \times \text{width} \times \text{height}$

$$V = 1/3 lwh$$

$$V = \frac{1}{3} \times 18 \text{ cm} \times 10 \text{ cm} \times 3\text{cm}$$

$$V = \frac{540 \text{ cm}^3}{3}$$

$$V = 180 \text{ cm}^3$$



Therefore, the volume of a rectangular pyramid is 180 cm³. The answer also can be written as 180 cubic centimeter.