DIVISION OF RADICALS AND RATIONALIZING DENOMINATORS

Division of Radicals

$$\frac{\sqrt[n]{15}}{\sqrt[n]{3}} = \sqrt[n]{5}$$

To divide radicals with the same index, we use the Division Property of Radicals.

DIVISION PROPERTY OF RADICALS

If a and b are nonnegative real numbers and $b \neq 0$.

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \text{ and } \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

A quotient of the n^{th} roots is the n^{th} root of the quotient of the radicands. If the index is the same, we just divide the radicands.

Example 1: Simplify each radical: A. $\frac{\sqrt{4}}{\sqrt{2}}$ B. $\frac{\sqrt{288}}{\sqrt{2}}$ C. $\sqrt{\frac{25}{9}}$ D. $\sqrt{\frac{3}{4}}$ E. $\sqrt{\frac{36}{9}}$ F. $\frac{\sqrt{26x^5}}{\sqrt{13x^3}}$

Solution: Apply Division Property.

A. $\frac{\sqrt{4}}{\sqrt{2}} = \sqrt{\frac{4}{2}} = \sqrt{2}$ 2 is not a perfect square. It remains under the radical sign.

B. $\frac{\sqrt{288}}{\sqrt{2}} = \sqrt{\frac{288}{2}} = \sqrt{144} = 12$ 144 is a perfect square.

C. $\sqrt{\frac{25}{9}} = \frac{\sqrt{25}}{\sqrt{9}} = \frac{5}{3}$ 25 and 9 are perfect squares.

D. $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$ 3, in the numerator, is not a perfect square. It remains under the radical sign. 4, in the denominator, is a perfect square.

E. $\sqrt{\frac{36}{9}} = \sqrt{4} = 2$ 4 is a perfect square.

F. $\frac{\sqrt{26x^5}}{\sqrt{13x^3}} = \sqrt{\frac{26x^5}{13x^3}} = \sqrt{2x^2} = x\sqrt{2}$ x^2 is a perfect square. 2 is not a perfect square. It remains under the radical sign.

Rationalizing Denominators

If the denominator of a fraction is not a rational number, we attempt to make it rational by getting rid of the radical in the denominator. It is called rationalizing the denominator.

RATIONALIZING DENOMINATORS

1. To rationalize the denominator of a fraction under the square root sign, multiply the numerator and denominator of the fraction by a quantity that makes the radicand of the denominator a perfect square.
2. To rationalize the denominator of a cube root, multiply by a radical that will produce a perfect cube under the cube root sign in the denominator.

Example 1: Rationalize the denominator of $\frac{\sqrt{5}}{\sqrt{2}}$.

Solution:

$$\frac{\sqrt{5}}{\sqrt{2}} = \frac{\sqrt{5}}{\sqrt{2}} \bullet \boxed{\frac{\sqrt{2}}{\sqrt{2}}}$$

Rationalize the denominator. Multiply the numerator and denominator by $\sqrt{2}$.

$$= \frac{\sqrt{5 \bullet 2}}{\sqrt{2 \bullet 2}}$$

$$= \frac{\sqrt{10}}{\sqrt{4}}$$

4 is a perfect square.

$$= \frac{\sqrt{10}}{2}$$

The denominator is now a rational number.

Example 2: Rationalize the denominator of $\sqrt{\frac{2}{3}}$.

Solution:

$$\sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}} \bullet \boxed{\frac{\sqrt{3}}{\sqrt{3}}}$$

Rationalize the denominator. Multiply the numerator and denominator by $\sqrt{3}$.

$$= \frac{\sqrt{6}}{\sqrt{9}}$$

9 is a perfect square.

$$= \frac{\sqrt{6}}{3}$$

The denominator is now a rational number.

Example 3: Rationalize the denominator of $\sqrt{\frac{5}{12x}}$.

Solution:

$$\sqrt{\frac{5}{12x}} = \frac{\sqrt{5}}{\sqrt{12x}} \bullet \boxed{\frac{\sqrt{3x}}{\sqrt{3x}}}$$

Rationalize the denominator. Multiply the numerator and denominator by $\sqrt{3x}$.

$$= \frac{\sqrt{15x}}{\sqrt{36x^2}}$$

36 and x^2 are perfect squares.

$$= \frac{\sqrt{15x}}{6x}$$

The denominator is now a rational number.

Example 4: Rationalize the denominator of $\frac{2\sqrt{5}}{\sqrt{6}}$.

Solution:

$$\frac{2\sqrt{5}}{\sqrt{6}} = \frac{2\sqrt{5}}{\sqrt{6}} \bullet \boxed{\frac{\sqrt{6}}{\sqrt{6}}}$$

Rationalize the denominator. Multiply the numerator and denominator by $\sqrt{6}$.

$$= \frac{2\sqrt{30}}{\sqrt{36}}$$

36 is a perfect square.

$$= \frac{2\sqrt{30}}{6}$$

$$= \frac{\overset{1}{\cancel{2}}\sqrt{30}}{\underset{3}{\cancel{6}}}$$

Simplify the fraction.

$$= \frac{\sqrt{30}}{3}$$

The denominator is now a rational number.

Example 5: Rationalize the denominator of $\frac{7}{\sqrt[3]{4}}$.

Solution:

$\frac{7}{\sqrt[3]{4}} = \frac{7}{\sqrt[3]{2^2}}$

4, in the denominator, is a perfect square, because the radical is a cube root, you will need a perfect cube instead of a square.

$= \frac{7}{\sqrt[3]{2^2}} \bullet \boxed{\frac{\sqrt[3]{2}}{\sqrt[3]{2}}}$

Multiply both numerator and denominator by $\sqrt[3]{2}$ and obtain $\sqrt[3]{2^3}$ in the denominator.

$= \frac{7\sqrt[3]{2}}{\sqrt[3]{2^3}}$

$= \frac{7\sqrt[3]{2}}{2}$

The denominator is now a rational number.

PRACTICE EXERCISES

Divide and simplify.

1. $\dfrac{2}{\sqrt{x}}$

2. $\dfrac{4}{\sqrt{8}}$

2. $\dfrac{5\sqrt{2}}{x^2\sqrt{3x}}$

3. $\sqrt{\dfrac{2}{3}}$

ANSWER KEY

1. $\dfrac{2\sqrt{x}}{\sqrt{x}}$

2. $\sqrt{2}$

3. $\dfrac{5\sqrt{6x}}{3x^3}$

4. $\dfrac{\sqrt{6}}{\sqrt{3}}$