Five Rules Of Positive Exponents

1) $a^m \bullet a^n$, 2) $\frac{a^m}{a^n}$, 3) $a^o = 1$, 4) $\left(a^m\right)^n$, 5) $\left(\frac{ax}{by}\right)^m$

Positive exponents have five rules. These rules apply only to multiplication and Division.

Rule 1: Product Rule for Exponent

$$a^m \bullet a^n = a^{m+n}$$

Example 1: Power with the same base:

(a). $x^8 \bullet x^4 = x^{8+4} = x^{12}$

(b). $y^7 \bullet y = y^{7+1} = y^8$ Note: $y = y^1$

(c). $x^4 \bullet y^2 =$ Cannot be simplified because the bases are different.

(d). $4^2 \bullet 4^4 = 4^{2+4} = 4^6 = 4096$

(e). $x^2 \bullet x^3 \bullet x^6 = x^{2+3+6} = x^{11}$ The rule applies to more than two factors.

(f). $6^m \bullet 6^n = 6^{m+n}$

(g). $x^{3n} \bullet x^{4n} = x^{3n+4n} = x^{7n}$

Rule 2: The Quotient of two Quantities with the Same Base

$$\frac{a^m}{a^n} = a^{m-n}, \qquad a \neq o$$

To find the quotient of two quantities with the same base, **subtract the powers**. If **a** is a nonzero real number and **m** and **n** are natural numbers:

Example 5: The same base raised to a power is the quotient.

(a). $\frac{x^4}{x^3} = x^{4-3}$ $\quad x = x^1$, the power 1 is not written.

$= x$

(b). $\frac{x^7}{x^7} = 1$

(c). $\frac{10^m}{10^n} = 10^{m-n}$

(d). $\frac{\left(a^2\right)^{15}}{a^6 a^4} = \frac{a^{30}}{a^{10}} = a^{30-10} = a^{20}$

(e). $\left(\frac{s^6}{s^3}\right)^4 = \left(s^{6-3}\right)^4 = \left(s^3\right)^4 = s^{12}$

(f). $\frac{b^5}{b^8} = \frac{1}{b^{8-5}} = \frac{1}{b^3}$

(g). $\frac{m^4}{n^2}$ Cannot be simplified because the base is different.

Rule 3: Zero Exponent Rule

$$\mathbf{a^{o} = 1, \qquad a \neq o}$$

When using the zero exponent rule, any real number, except 0, raised to the zero power equals 1. Remember $\mathbf{0^{0}}$ is undefined.

Example 3 When $\mathbf{a \neq o}$.

(a). $\mathbf{6^{0}} = \mathbf{1}$

(b). $\mathbf{(-3)^{0}} = \mathbf{1}$

(c). $\mathbf{2x^{0}} = \mathbf{2(1)}$

(d). $\mathbf{(-7x)^{0}} = \mathbf{1}$

(e). $\mathbf{-5^{0} + (-5)^{0}}$

$\mathbf{-1 + 1} = \mathbf{0}$

(f). $\mathbf{9x^{3}y^{4}z^{0}} = \mathbf{9x^{3}y^{4}(1)}$

$= \mathbf{9x^{3}y^{4}}$

Rule 4: Power rule for Exponents

$$\left(a^{m}\right)^{n} = a^{m \bullet n} = a^{mn}$$

When a quantity is raised to another power, multiply the exponents. That is, if **m** and **n** are natural numbers and **a** is a real number:

Example 2: Power to another power:

(a). $\left(x^{3}\right)^{4} = x^{3 \bullet 4} = x^{12}$

(b). $\left(y^{8}\right)^{3} = y^{8 \bullet 3} = y^{24}$

(c). $\left(2^{x}\right)^{y} = 2^{x \bullet y} = 2^{xy}$

(d). $(3m)^{2} = 3m^{1 \bullet 2} = 3m^{2}$

$(3m)^{2} = 3^{2}m^{1 \bullet 2}$

$= 9m^{2}$

Rule 5: Expanded Power Rule for Exponents

$$\left(\frac{ax}{by}\right)^m = \frac{a^m x^m}{b^m y^m}, \qquad b \neq 0,\ y \neq 0$$

Each factor in the parentheses is raised to a power. If **m** and **n** are natural numbers and **a** is a real number:

Example 3: Product raised to a power:

(a). $(mn)^5 = m^5 n^5$

(b). $(3x)^3 = 3^3 x^{1 \bullet 3}$ — 3^3 can be simplified to its lowest terms.

$= 27x^3$

(c). $10(-2rs)^4 = 10(-2)^4 r^4 s^4$

$= 10(16)r^4 s^4$

$= 160r^4 s^4$

(d). $2x^4(3x)^3 = 2x^4 \bullet 3^3 \bullet x^3$

$= 2\left(3^3\right)x^4 \bullet x^3$

$= 2(27)x^7$

$= 54x^7$

(e). $\left(2x^2 y^4 z\right)^2 = 2^2\left(x^2\right)^2\left(y^4\right)^2\left(z^1\right)^2$ — 2^2 can be simplified to its lowest term.

$= 4x^4 y^8 z^2$

(f). $\left(\frac{x}{4}\right)^2 = \frac{x^2}{4^2}$

$= \frac{x^2}{16}$