Division of a Polynomial by a Monomial

$$\frac{4x^2 + 3x^4 + 6x^6}{2x}$$

If a, b, and c are real numbers, then $\frac{a+b+\ldots}{c} = \frac{a}{c} + \frac{b}{c}$ **$c \neq 0$**

A **polynomial** is a variable expression in which the terms are monomials.

$$2x^3, -4x^2, -3$$

A **monomial** is a number, a variable, or a product of numbers and variables.

Polynomial terms can be computed in the form of addition, subtraction, multiplication or division. When dividing a polynomial by a monomial, there are two rules to complete.

Rule 1- Division of Polynomial

1. **To divide a polynomial by a monomial, write the division as a fraction. "Split the numerators," or "Split the fraction."**
2. **Divide each term in the polynomial (dividend) by the monomial (divisor).**
3. **Use the Division of Exponential Expression rule:** $\frac{x^a}{x^b} = x^{a-b}$

Rule 2- Division of Exponential Expression

If a and b are integers and $x \neq 0$, then $\frac{x^a}{x^b} = x^{a-b}$ **if $a > b$**

and $\frac{x^a}{x^b} = \frac{1}{x^{b-a}}$ **if $a < b$**

Note: Remember to subtract the variables.

Example 1: Divide the polynomial $6x^4 + 2x^5 - 4x$ by $-2x^2$

Solution:

Step 1 Rewrite the division as a fraction.

$$\frac{6x^4 + 2x^5 - 4x}{-2x^2}$$

Step 2 "Split the numerator," or use the monomial as the denominator.

$$\frac{6x^4}{-2x^2} + \frac{2x^5}{-2x^2} - \frac{4x}{-2x^2}$$

Step 3 Divide each term in the polynomial by the monomial.
(Rule # 2: Subtract the variables.)

$$= -\frac{6x^4}{2x^2} - \frac{2x^5}{2x^2} + \frac{4x}{2x^2}$$ ← Signs change

$$= -3x^{4-2} - 1x^{5-2} + 2x^{1-2}$$ ← Rule # 2: Subtract the variables. Note: $2x^{1-2}$, variable answer went in the denominator.

$$= -3x^2 - 1x^3 + \frac{2}{x}$$

Therefore $\frac{6x^4 + 2x^5 - 4x}{-2x^2} = -3x^2 - 1x^3 + \frac{2}{x}$

Example 2: Divide $\frac{16m^4 + 8m^5 - 4m}{-4m}$

Solution:

Step 1 Write the division as a fraction.

$$\frac{16m^4 + 8m^5 - 4m}{-4m}$$

Step 2 "Split the numerator," or use the monomial as the denominator.

$$\frac{16m^4}{-4m} + \frac{8m^5}{-4m} - \frac{-4m}{-4m}$$

Step 3 Divide each term in the polynomial by the monomial.
(Rule # 2: Subtract the variables.)

$$\frac{-16m^4}{4m} - \frac{8m^5}{4m} + \frac{4m}{4m}$$ ← Signs change

$$= -4m^{4-1} - 2m^{5-1} + 1m^{1+1}$$ ←Rule #2 subtract the variables.

$$= -4m^3 - 2m^4 + 1$$

Therefore, $\frac{16m^4 + 8m^5 - 4m}{-4m} = 4m^3 - 2m^4 + 1$

Example 3: Divide $3x^6 + 18x^4 - 6x + 2x^3 \div 2x$

Solution:

Step 1 Write the division as a fraction.

$$\frac{3x^6 + 18x^4 - 6x + 2x^3}{2x}$$

Step 2 "Split the numerator," or use the monomial as the denominator.

$$\frac{3x^6}{2x} + \frac{18x^4}{2x} - \frac{6x}{2x} + \frac{2x^3}{2x}$$

Step 3 Divide each term in the polynomial by the monomial.
(Rule # 2: Subtract the variables.)

$$= \frac{3x^6}{2x} + \frac{18x^4}{2x} - \frac{6x}{2x} + \frac{2x^3}{2x}$$

$$= \frac{3x^{6-1}}{2} + 9x^{4-1} - 3 + x^{3-1}$$ ←Rule #2: Subtract the variables.

$$= \frac{3x^5}{2} + 9x^3 - 3 + x^2$$ ←Note: $\frac{2x^3}{2x} = 1x^2$, written as x^2.

Therefore, $$\frac{3x^6 + 18x^4 - 6x + 2x^3}{2x} = \frac{3x^5}{2} + 9x^3 - 3 + x^2$$

Example 4: Divide $\frac{6a^4b - 4a^7b^6 + 10a^5b^3}{-2a^2b}$

Solution:

Step 1 Write the division as a fraction.

$$\frac{6a^4b - 4a^7b^6 + 10a^5b^3}{-2a^2b}$$

Step 2 "Split the numerator, " or use the monomial as the denominator.

$$\frac{6a^4b}{-2a^2b} - \frac{4a^7b^6}{2a^2b} + \frac{10a^5b^3}{-2a^2b}$$

Step 3 Divide each term in the polynomial by the monomial.
(Rule # 2: Subtract the variables.)

$$= \frac{-6a^4b}{2a^2b} + \frac{4a^7b^6}{2a^2b} - \frac{10a^5b^3}{2a^2b}$$ ←Signs change

$$= -3a^{4-2}b^{1-1} + 2a^{7-2}b^{6-1} - 5a^{5-2}b^{3-1}$$ ←Rule #2: Subtract the variables.

$$= -3a^2 + 2a^5b^5 - 5a^3b^2$$

Therefore, $\frac{6a^4b - 4a^7b^6 + 10a^5b^3}{-2a^2b} = -3a^2 + 2a^5b^5 - 5a^3b^2$

Example 5: Divide $\left(6y^4 - 9y^2 + 7y - 2\right) \div -3y$

Solution:

Step 1 Write the division as a fraction.

$$\frac{6y^4 - 9y^2 + 7y - 2}{-3y}$$

Note: To clear the negative in the denominator, multiply the numerator and denominator by -1 to get a positive denominator.

$$\frac{(-1)\left(6y^4 - 9y^2 + 7y - 2\right)}{(-1)(-37)} = \frac{-6y^4 = 9y^2 - 7y + 2}{3y}$$

Step 2 "Split the numerator," or use the monomial as the denominator.

$$= \quad \frac{-6y^4}{3y} \quad + \quad \frac{9y^2}{3y} \quad - \quad \frac{7y}{3y} \quad + \quad \frac{2}{3y}$$

Step 3 Divide each term in the polynomial by the monomial. (Rule # 2: Subtract the variables.)

$$= \quad \frac{-6y^4}{3y} \quad + \quad \frac{9y^2}{3y} \quad - \quad \frac{7y}{3y} \quad + \quad \frac{2}{3y}$$

$$= \quad -2y^{4-1} \quad + \quad 3y^{2-1} \quad - \quad \frac{7y^{1-1}}{3} \quad + \quad \frac{2}{3y}$$ ←Rule #2: Subtract the variables.

$$= \quad -2y^3 \quad + \quad 3y^2 \quad - \quad \frac{7}{3} \quad + \quad \frac{2}{3y}$$

Therefore, $\dfrac{-6y^4 + 9y^2 - 7y + 2}{3y} = -2y^3 + 3y^2 - \dfrac{7}{3} + \dfrac{2}{3y}$

Learning Assistance has the following resources available for <u>Division of polynomials</u>:

1. CD-ROM #1, lecture series to accompany <u>Elementary Algebra</u>, 6th Ed.
2. Videotape # 6, <u>Basic College Algebra</u> by Gerald Davis