

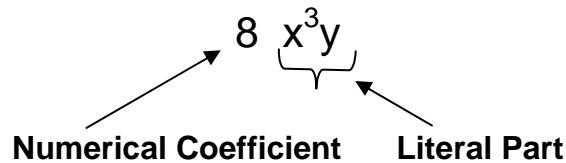


ADDITION AND SUBTRACTION OF POLYNOMIALS

A handout from The Learning Center at Trident Technical College

Polynomial: An algebraic expression consisting of the Sum (+) and the Difference (−) of one or more terms.

The addition and subtraction in an algebraic expression is divided into parts called **terms**. The **number** in a term is called **the numerical coefficient**, and **the variables** are called **the literal parts**.



Two terms are called **like terms** if they differ only in their **numerical coefficient**. **Like terms** must have exactly the same **literal parts** (the same variables and the same exponents) to add or subtract.

$$8x^3y, \quad -4x^3y$$

Like Terms

$$4x^2y^2, \quad -6x^3y^2$$

Unlike Terms

Polynomials can be added or subtracted, using either a **horizontal or vertical format**, by **combining the like terms**.

To add polynomials horizontally:

1. Remove the polynomials out of the parenthesis.
2. Write the polynomials in descending order.
3. Combine all the like terms occurring in the polynomials being added.

EXAMPLE 1: Add $(3x^2 + 3x - 1) + (4x^2 - 6x - 5)$

Solution

$$(3x^2 + 3x - 1) + (4x^2 - 6x - 5)$$

Remove terms from the parentheses.

$$= 3x^2 + 3x - 1 + 4x^2 - 6x - 5$$

$$= \underbrace{3x^2 + 4x^2} + \underbrace{3x - 6x} - \underbrace{1 - 5}$$

Group like terms in descending order.

$$= 7x^2 - 3x - 6$$

Combine like terms.

EXAMPLE 2: Add $(5t^5 + 3t^3 - 1) + (7t^3 - 8t^2 + 14)$

Solution

$$(5t^5 + 3t^3 - 1) + (7t^3 - 8t^2 + 14)$$

Remove the terms from the parentheses.

$$= 5t^5 + 3t^3 - 1 + 7t^3 - 8t^2 + 14$$

$$= \underbrace{5t^5} + \underbrace{3t^3 + 7t^3} - \underbrace{8t^2} - \underbrace{1 + 14}$$

Group like terms in descending order.

$$= 5t^5 + 10t^3 - 8t^2 + 13$$

Combine like terms.

To add polynomials vertically:

1. Write the polynomials in descending order with like terms under one another.
2. Combine like terms.

EXAMPLE 3: Add $6a^2 + 3a - 1$ and $4a^2 - 6a - 5$

Solution

$$6a^2 + 3a - 1 \text{ and } 4a^2 - 6a - 5$$

$$\begin{array}{r} 6a^2 + 3a - 1 \\ + 4a^2 - 6a - 5 \\ \hline 10a^2 - 3a - 6 \end{array}$$

Arrange like terms in columns.

Combine like terms.

EXAMPLE 4: Add $4x^3 + 6x^4 - 3x^2 + 1$ and $-4 + 3x^3 + 2x^2 + 3x$

Be very careful to line up the columns!

Solution

$$\begin{array}{r} 6x^4 + 4x^3 - 3x^2 + 1 \\ + \quad \quad + 3x^3 + 2x^2 + 3x - 4 \\ \hline 6x^4 + 7x^3 - x^2 + 3x - 3 \end{array}$$

Arrange like terms under one another in a column.

Combine like terms.

SUBTRACTION OF POLYNOMIALS

To subtract polynomials, remove terms in a parentheses preceded by a negative sign of each term in the parentheses. (**Hint:** Another method is to multiply by -1 . The distributive property allows this.)

EXAMPLE 5: Subtract $(4x^3 - 3x^2 - 7) - (7x^3 - 4x^2 - 2)$

Solution:

$$(4x^3 - 3x^2 - 7) - (7x^3 - 4x^2 - 2)$$

$$-7x^3 + 4x^2 + 2$$

EXAMPLE 6: Subtract $(4x^3 - 3x^2 - 7) - (7x^3 - 4x^2 - 2)$

Solution:

$$(4x^3 - 3x^2 - 7) - 1(7x^3 - 4x^2 - 2)$$

$$-7x^3 + 4x^2 + 2$$

To subtract polynomials horizontally:

1. Write each polynomial in descending order.
2. Change the sign of each term of the polynomial being subtracted.
3. Combine like terms.

EXAMPLE 7: Subtract $(8y^3 + 2y^2 + 6y - 8) - (-3y^3 - 8y^2 + 2y - 8)$

Solution

$$(8y^3 + 2y^2 + 6y - 8) - (-3y^3 - 8y^2 + 2y - 8)$$

Remove terms from the parentheses in descending order

$$= 8y^3 + 2y^2 + 6y - 8 + 3y^3 + 8y^2 - 2y + 8$$

Change the sign of each term of the polynomial being subtracted.

$$= \underbrace{8y^3 + 3y^3} + \underbrace{2y^2 + 8y^2} + \underbrace{6y - 2y - 8 + 8}$$

Group like terms.

$$= 11y^3 + 10y^2 + 4y$$

Combine terms.

EXAMPLE 8: Subtract $(y^2 - 10xy) - (2y^2 + 3xy)$

Solution:

$$(y^2 - 10xy) - (2y^2 + 3xy)$$

$$= y^2 - 10xy - 2y^2 - 3xy$$

Remove the terms from the parentheses.

$$= y^2 - 10xy - 2y^2 - 3xy$$

Change the sign of each term of the polynomial being subtracted.

$$= \underbrace{y^2 - 2y^2} - \underbrace{10xy - 3xy}$$

Group terms.

$$= -1y^2 - 13xy$$

Combine like terms.

To subtract polynomials vertically:

1. Write each polynomial in descending order, with like terms in the same column.
2. Change the sign of each term of the polynomial being subtracted.
3. Combine like terms.

EXAMPLE 9: Subtract $(12m^3 - 4m^2 - 10m - 5) - (3m^3 - 5m^2 - 6m - 3)$

Solution:

$$\begin{array}{r} 12m^3 - 4m^2 - 10m - 5 \\ -(3m^3 - 5m^2 - 6m - 3) \\ \hline \end{array}$$

Write each polynomial in descending order with like terms in the same column.

$$= \begin{array}{r} 12m^3 - 4m^2 - 10m - 5 \\ + (-3m^3 - 5m^2 - 6m - 3) \\ \hline \end{array}$$

Change the sign of each term of the polynomial being subtracted.

$$= \begin{array}{r} 12m^3 - 4m^2 - 10m - 5 \\ + (-3m^3 + 5m^2 + 6m + 3) \\ \hline \\ = \frac{9m^3 + 1m^2 - 4m - 2}{} \end{array}$$

Combine like terms.

EXAMPLE 10: Subtract: $(8y^7 - 2y^3 + 2y^2 - 5) - (-2y^7 + 2y^2 + 4)$

Solution: Use the vertical format.

$$(8y^7 - 2y^3 + 2y^2 - 5) - (-2y^7 + 2y^2 + 4)$$

$$= \begin{array}{r} 8y^7 - 2y^3 + 2y^2 - 5 \\ - \begin{array}{r} \overset{(+)}{-2y^7} \quad \overset{(-)}{+2y^2} \quad \overset{(-)}{+4} \end{array} \\ \hline \end{array}$$

$$= \begin{array}{r} 8y^7 - 2y^3 + 2y^2 - 5 \\ + \quad 2y^7 \quad \quad - 2y^2 - 4 \\ \hline 10y^7 - 2y^3 \quad \quad - 9 \end{array}$$

Arrange like terms in columns.

Leave space for missing term.
Change sign of each term of the polynomial being subtracted.

Combine like terms.

ADDITION AND SUBTRACTION PRACTICE

Add or subtract as indicated:

1. Add:
$$\begin{array}{r} 4x^5 \quad - 6x^2 + 11 \\ + \quad - 2x^4 \quad - 6x^2 - 5 \\ \hline \end{array}$$

2. Add: $(3r^3 + 5r^2 - 6) + (2r^3 - 6r^2 - 3)$

3. Subtract: $(-2b^2 + 3b - 5) - (-b^2 + 2b + 3)$

4. Subtract:
$$\begin{array}{r} 5a^4 - 3a^3 + 2a^2 - 5a + 6 \\ - \quad (-a^3 - a^2 - 2a - 1) \\ \hline \end{array}$$

BONUS: $(3X^5 + 2X^2 + 5) - (-7X^5 - 8X^2 + 2) + (3X^2 - 4X + 7)$

ANSWER:

1. $4x^5 - 2x^4 - 12x + 6$

2. $5r^3 - 1r^2 - 9$

3. $-1b^2 + 1b - 8$

4. $5a^4 - 2a^3 + 3a^2 - 3a + 7$

BONUS

5. $10x^5 + 13x^2 - 4x + 10$

